Review 24 Improper Integrals

$\int_0^\infty xe^{-x}\,dx = 1$ infinity as a limit of integration

make it proper

$\lim_{b\to\infty}\int_0^b xe^{-x}\,dx = \lim_{b\to\infty}\left(-xe^{-x}-e^{-x}\Big|_0^b\right)$

$x \quad +e^{-x}$

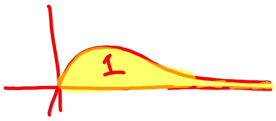
$1 \quad -e^{-x}$

$0 \quad e^{-x}$

$= \lim_{b\to\infty}\left(-be^{-b}-e^{-b}\right)-\left(0-e^{0}\right)$

$\lim_{b\to\infty}\left(\frac{-b}{e^b}-\frac{1}{e^b}\right)+1 = 1$

$\frac{-b}{e^b}\to 0 \qquad \frac{1}{e^b}\to 0$



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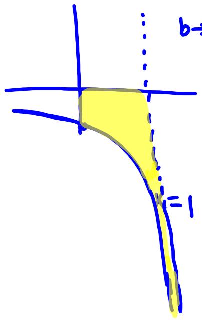
diverges

$\int_0^1 \frac{1}{x-1}\,dx$ undefined at an endpoint

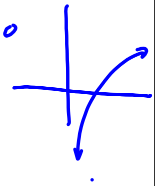


$\lim_{b\to 1^-}\int_0^b \frac{1}{x-1}\,dx = \lim_{b\to 1^-} \ln|x-1|\Big|_0^b$

$= \lim_{b\to 1^-} \ln|b-1| - \ln|0-1|$

$= -\infty - 0$

$= -\infty$



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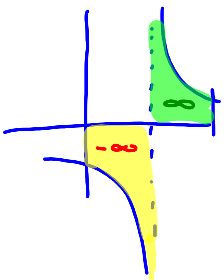
diverges

$\int_0^2 \frac{1}{x-1}\,dx$ undefined at an interior point

$\int_0^1 \frac{1}{x-1}\,dx + \int_1^2 \frac{1}{x-1}\,dx$

we just showed this diverges



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$0 \le \int_1^\infty \frac{1}{x^3+1}\,dx \le \int_1^\infty \frac{1}{x^3}\,dx = \frac{1}{2}$

converges by comparison

$\lim_{b\to\infty}\int_1^b x^{-3}\,dx$

$\lim_{b\to\infty}\frac{x^{-2}}{-2}\Big|_1^b = \lim_{b\to\infty}\left(\frac{1}{-2b^2}-\frac{1}{-2\cdot 1^2}\right) = \frac{1}{2}$

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$f(x) \geq g(x) \geq 0$

if $\int_a^\infty f(x)\,dx$ converges then $\int_a^\infty g(x)\,dx$ converges

if $\int_a^\infty g(x)\,dx$ diverges then $\int_a^\infty f(x)\,dx$ diverges

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